1) Show that $y = x - x^{-1}$ is a solution of the differential equation $xy' + y = 2x$.

$\boxed{\text{Show}}$

2) Verify that $y = \sin x \cos x - \cos x$ is a solution of the initial-value problem:

$$y' + (\tan x)\, y = \cos^2 x \qquad y(0) = -1$$

on the interval $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

$\boxed{\text{Show}}$

3) For what nonzero values of k does the function $y = \sin kt$ satisfy the differential equation $y'' + 9y = 0$? For those values of k, verify that every member of the family of functions $y = A\sin kt + B\cos kt$ is also a solution.

$\boxed{\begin{array}{l} k = \pm 3 \\ \text{Show} \end{array}}$

4) For what values of r does the function $y = e^{rt}$ satisfy the differential equation $y'' + y' - 6y = 0$?

$$\boxed{r = -3 \text{ or } 2}$$

5) A population is modeled by the differential equation:

$$\frac{dP}{dt} = 1.2P\left(1 - \frac{P}{4200}\right)$$

a) For what values of P is the population increasing? $\boxed{0 < P < 4200}$

b) For what values of P is the population decreasing? $\boxed{P > 4200}$

c) What are the equilibrium solutions? $\boxed{P = 4200 \text{ or } P = 0}$

6) A function $y(t)$ satisfies the differential equation:

$$\frac{dy}{dt} = y^4 - 6y^3 + 5y^2$$

a) What are the constant solutions of the equation? $\boxed{k = 0,\ 1,\ \text{or } 5}$

b) For what values of y is y increasing? $\boxed{y \in (-\infty, 0) \cup (0,1) \cup (5, \infty)}$

c) For what values of y is y decreasing? $\boxed{y \in (1,5)}$